

1. **Radiative corrections to $m_{h^0}^2$**

At tree level in the MSSM the lightest Higgs mass is bounded above, $m_{h^0}^2 \leq m_Z^2$. Loop corrections weaken this bound. The most important corrections come from top quark loops.

Some background: terms in the superpotential $\lambda_u^{ij} Q_i \bar{u}_j H_u$ lead to a coupling between the (Dirac spinor) top quark t and the (real scalar) neutral up-type Higgs h_u^0 ,

$$\mathcal{L} = y_t h_u^0 \bar{t} t + \dots$$

Here y_t is the top Yukawa coupling. Electroweak symmetry breaking leads to

$$h_u^0 = \frac{1}{\sqrt{2}}(v_u + \hat{h}_u)$$

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \hat{h}_u \\ \hat{h}_d \end{pmatrix}$$

where $v_u = v \sin \beta$, $v \approx 246 \text{ GeV}$ and α is a mixing angle (see for example Martin (7.35)). Let's suppose that $m_H^2, m_A^2 \gg m_h^2, m_Z^2$ so that $\alpha \approx \beta - \pi/2$ (see Martin p. 64).

- (i) At energies below the stop mass $m_{\bar{t}}$ the dynamics of the light Higgs field h can be described by an effective Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} \mu^2 h^2 - \frac{1}{4} \lambda h^4 + \bar{t}(i\not{\partial} - m_t)t - gh\bar{t}t. \quad (1)$$

where $g = m_t/v$. Compute the correction to the coupling λ from a loop of top quarks

You should Wick rotate to Euclidean space and put in a momentum cutoff $|k_E| < m_{\bar{t}}$. (This takes into account the fact that above the scale $m_{\bar{t}}$ stop loops will cancel the top loop.) Also you only need to keep the leading log behavior of the integral for $m_t \ll m_{\bar{t}}$.

- (ii) The Higgs mass is $m_h^2 = \lambda v^2$, as you can see by expanding the Higgs potential in (1) about its minimum. So the correction to the coupling you've computed gives a correction to the Higgs mass, $\delta m_h^2 = \delta \lambda v^2$. Express δm_h^2 in terms of m_t , $m_{\tilde{t}}$, v .
- (iii) For $m_{\tilde{t}} = 500 \text{ GeV}$, what's the correction?

2. Charge and color breaking minima

In this problem we'll study a particular direction in field space, focusing on the up-type Higgs doublet h_u , the up-type squark doublet \tilde{q}_u , and the up-type squark singlet \tilde{u} . These fields have $SU(3) \times SU(2) \times U(1)$ quantum numbers $(\mathbf{1}, \mathbf{2}, 1)$, $(\mathbf{3}, \mathbf{2}, 1/3)$, $(\bar{\mathbf{3}}, \mathbf{1}, -4/3)$. You can set all other MSSM scalars to zero.

- (i) Consider configurations where these fields have equal but otherwise arbitrary magnitudes: $|h_u| = |\tilde{q}_u| = |\tilde{u}|$. Show that the D -flatness conditions can be satisfied by making the fields point in the right directions in the gauge group representation space. (It might help to recall some of the manipulations we did in class on the D -term part of the Higgs potential.)
- (ii) Evaluate the F -term part of MSSM potential for these D -flat field configurations. Show that the F -terms generate a potential that keeps the fields at the origin.
- (iii) Finally add the soft susy breaking terms and evaluate the full MSSM potential.
- (iv) The cubic coupling $A_u \tilde{q}_u \tilde{u} h_u$ can give rise to a minimum away from the origin. Suppose we require that the global minimum is at the origin. Use this to derive a bound on A_u .

Moral of the story: depending on the soft breaking parameters the MSSM potential can have unwanted minima that (in this case) spontaneously break color and $U(1)_{\text{em}}$ gauge symmetry.