

1. Super-Higgs mechanism

Recall that massless super-QED is described by the action

$$S = \int d^4x d^4\theta (\Phi_+^\dagger e^{eV} \Phi_+ + \Phi_-^\dagger e^{-eV} \Phi_-) + \left[\int d^4x d^2\theta \frac{1}{4} W^\alpha W_\alpha + \text{c.c.} \right]$$

for an abelian vector multiplet V and a pair of chiral multiplets Φ_+ , Φ_- . This theory has a moduli space of vacua which spontaneously break the $U(1)$ gauge symmetry (except at the special point $\phi_+ = \phi_- = 0$).

To see the consequences of this symmetry breaking set

$$\Phi_+ = e^{ie\Lambda} X \quad \Phi_- = e^{-ie\Lambda} X$$

in terms of two new chiral multiplets Λ , X .¹ Go to unitary gauge by setting $\Lambda = 0$; note that this means we're no longer free to use Wess-Zumino gauge. Expand the action to quadratic order in superfields and show that when the bottom component of X is non-zero the gauge field A_μ acquires a mass term.

Moral of the story: in the super-Higgs mechanism a vector multiplet eats a chiral multiplet. In the case of massless super-QED one ends up with a massless chiral multiplet X coupled to a massive vector multiplet V .

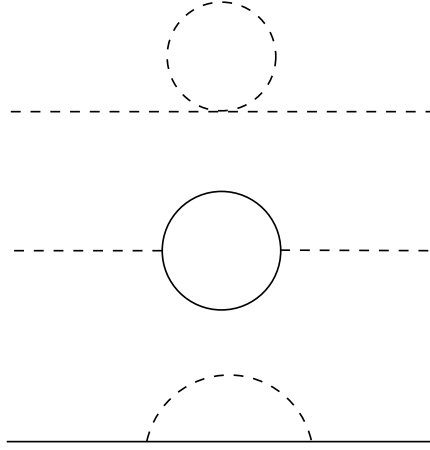
2. Supersymmetry and the hierarchy problem

Consider a WZ model with a single chiral superfield Φ and a superpotential $W = \frac{1}{3}g\Phi^3$.

- (i) Write out the Lagrangian \mathcal{L} for the physical degrees of freedom ϕ and ψ . Determine the propagators and vertices for these fields. (You can obtain the ψ propagator from i times the inverse of the operator that appears in the Lagrangian – just as you would for a Dirac field).

¹Strictly speaking to get a 1 : 1 change of variables we should identify X with $-X$. For our present purposes this doesn't matter much.

- (ii) Compute the corrections to the scalar and fermion propagators coming from the diagrams



The dashed line is the scalar propagator, the solid line is the fermion propagator. You can work at zero external momentum, with a cutoff Λ on the Euclidean loop momentum.

- (iii) At tree level the fields ϕ and ψ are massless. According to your results in part (ii), do they remain massless when one-loop radiative corrections are taken into account?
- (iv) Consider adding a mass term for the scalar field to the WZ Lagrangian, $\mathcal{L} \rightarrow \mathcal{L} - m^2 \phi^* \phi$. Note that the mass term explicitly breaks supersymmetry. Now what are the one-loop corrections to the scalar and fermion masses?
- (v) Consider changing the value of the quartic scalar coupling, $\mathcal{L} \rightarrow \mathcal{L} - \lambda(\phi^* \phi)^2$. Again this modification explicitly breaks supersymmetry. What happens to the one-loop mass corrections?
- (vi) Show that the Lagrangian with $m, \lambda \neq 0$ has a chiral $U(1)$ symmetry $\psi \rightarrow e^{-i\chi/3} \psi, \phi \rightarrow e^{i2\chi/3} \phi$. Show that this symmetry forbids a mass term for the fermion. Show that when $m, \lambda = 0$ this symmetry becomes an R -symmetry that forbids a supersymmetric mass term for Φ .

Moral of the story: fermion masses can be prohibited by chiral symmetry. If supersymmetry is unbroken then scalar masses are also forbidden. If supersymmetry is “softly broken” as in part (iv) the masses pick up log divergent corrections. But “hard breaking” as in part (v) leads to quadratically divergent scalar masses.